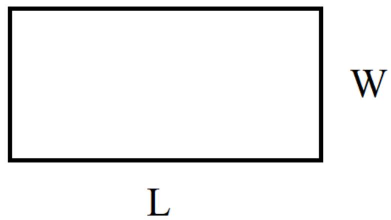
Exercise 85

Find the dimensions of the rectangular corral producing the greatest enclosed area given 200 feet of fencing.

Solution

Draw a schematic of the rectangular corral, labelling the length and width as L and W, respectively.



The perimeter is the sum of the rectangle's sides.

$$\begin{aligned} P &= L + L + W + W \\ &= 2L + 2W \end{aligned}$$

It's given to be 200 feet.

$$200 = 2L + 2W$$

Solve for L.

$$200 - 2W = 2L$$

$$\frac{1}{2}(200 - 2W) = L$$

$$L = 100 - W$$

Write the formula for the area, substitute the result for the length, and complete the square to write the quadratic function in vertex form.

$$\begin{aligned} A &= LW \\ &= (100 - W)W \\ &= 100W - W^2 \\ &= -(W^2 - 100W) \\ &= -[(W^2 - 100W + 50^2) - 50^2] \\ &= -[(W - 50)^2 - 50^2] \\ &= -(W - 50)^2 + 50^2 \end{aligned}$$

Therefore, the maximum area is $A = 50^2 = 2500$ ft^2, which occurs when $W = 50$ ft and $L = 100 - 50 = 50$ ft.